A NON-STANDARD INTEGRAL EQUATION WITH APPLICATIONS TO QUASICONFORMAL MAPPINGS

BY

LIPMAN BERS

Columbia University, New York, N.Y., U.S.A. (1)

1. Statement of results

A) Let C denote an oriented closed Jordan curve in the extended complex plane (Riemann sphere). We denote by $D_1=D_1(C)$ and $D_2=D_2(C)$ the domains interior and exterior to C, respectively, and we denote by $\lambda_j(z)|dz| =\lambda_{D_j(C)}(z)|dz|$ the Poincaré metric in D_j, $j=1, 2$. We denote by q an integer, $q\geqslant 2$.

In this paper we investigate the integral equation

$$\varphi(z)=\iint_{D_1}(\zeta-z)^{-2q}\lambda_1(\zeta)^{2-2q}\overline{\psi(\zeta)}\,d\xi\,d\eta, \quad z\in D_2, \tag{1.1}$$

where the given function φ and the unknown function ψ are assumed to be holomorphic in D_2 and D_1, respectively. We will give conditions under which the equation is uniquely solvable, and some applications of these conditions.

We write equation (1.1) in the abbreviated form

$$\varphi=\mathfrak{L}_C^{(q)}\psi, \tag{1.2}$$

and denote by $-C$ the curve C with the orientation reversed. Thus

$$\varphi=\mathfrak{L}_{-C}^{(q)}\psi$$

is an abbreviation for the formally transposed equation

$$\varphi(z)=\iint_{D_2}(\zeta-z)^{-2q}\lambda_2(\zeta)^{2-2q}\overline{\psi(\zeta)}\,d\xi\,d\eta, \quad z\in D_1.$$

---

(1) Work supported by NSF (Contract No. GP 3433). Some results have been announced in [5].