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The Wiener test and potential estimates for quasilinear elliptic equations

by

TERO KILPELÄINEN and JAN MALÝ

University of Jyväskylä
Jyväskylä, Finland

Charles University
Prague, The Czech Republic

1. Introduction

Let Ω be an open set in $\mathbf{R}^n$ and let $1<p\leqslant n$ be a fixed number. Consider the quasilinear partial differential operator

$$Tu=-\operatorname{div}\mathcal{A}(x,\nabla u),$$

where $u\in W^{1,p}_{\text{loc}}(\Omega)$ and $\mathcal{A}(x,\xi)\cdot\xi\approx|\xi|^p$; the precise assumptions on $\mathcal{A}$ are listed in Section 2. The principal model operator is the p-Laplacian

$$Tu=-\Delta_p u=-\operatorname{div}(|\nabla u|^{p-2}\nabla u),$$

and so the ordinary Laplacian $\Delta=\Delta_2$ is included in our study.

A boundary point x_0 of bounded Ω is *regular* if the solution u to the Dirichlet problem

$$\begin{cases} Tu=0 & \text{in } \Omega \\ u-f\in W^{1,p}_0(\Omega) \end{cases}$$

has the limit value $f(x_0)$ at x_0 whenever $f\in W^{1,p}(\Omega)$ is continuous in the closure of Ω. In [23] Wiener proved that in the case of the Laplacian the regularity of a boundary point $x_0\in\partial\Omega$ can be characterized by a so called Wiener test, where one measures the thickness of the complement of Ω near x_0 in terms of capacity densities; we soon come to the precise formulation of this test. In the fundamental work [17] Littman, Stampacchia, and Weinberger showed that the same Wiener test identifies the regular boundary points whenever T is a uniformly elliptic linear operator with bounded measurable coefficients; then the regularity of a boundary point is independent of the particular operator.

For general nonlinear operators the classical Wiener test has to be modified so that the type p of the operator T is involved. Maz'ya [18] established in 1970 that the