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Precise damping conditions for global asymptotic stability for nonlinear second order systems

by

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1. Introduction

The global asymptotic stability of the rest point for nonlinear equations has been treated by Levin and Nohel, Salvadori, Thurston and Wong, Artstein and Infante, and Ballieu and Peiffer. These studies have been generalized to scalar variational problems in [8] and to variational systems in [9]. Here we shall unify and further extend this work, by obtaining necessary and sufficient conditions for the asymptotic stability of solutions of quasi-variational systems in terms of the damping functions of the systems treated.

Throughout the paper we thus consider vector unknowns $u: J\to\mathbf{R}^N$, where J is a half open interval of the form $[R,\infty)$. The typical system which we shall study then has the form

$$(\nabla\mathcal{L}(t,u,u'))'-\nabla_u\mathcal{L}(t,u,u')=Q(t,u,u'), \tag{1.1}$$

where $\mathcal{L}(t,u,p)=G(u,p)-F(t,u)$ and where

$$G:\mathbf{R}^N\times\mathbf{R}^N\to\mathbf{R},\quad F:J\times\mathbf{R}^N\to\mathbf{R},\quad Q:J\times\mathbf{R}^N\times\mathbf{R}^N\to\mathbf{R}^N$$

are given continuously differentiable functions. The most important of the conditions which will be imposed on (1.1) are that

$$G(u,\cdot)\text{ is strictly convex in }\mathbf{R}^N;\quad G(u,0)=0,\quad \nabla G(u,0)=0, \tag{1.2}$$

$$(\nabla_u F(t,u),u)>0\quad\text{for } u\neq 0;\quad F(t,0)=0, \tag{1.3}$$

$$(Q(t,u,p),p)\leqslant 0. \tag{1.4}$$

Here $(\cdot,\cdot)$ denotes the inner product in $\mathbf{R}^N$ and

$$\nabla=\left(\frac{\partial}{\partial p_1},\dots,\frac{\partial}{\partial p_N}\right),\quad \nabla_u=\left(\frac{\partial}{\partial u_1},\dots,\frac{\partial}{\partial u_N}\right).$$