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# A criterion of algebraicity for Lelong classes and analytic sets

by

AHMED ZERIAHI

*Université Paul Sabatier*
*Toulouse, France*

## 1. Introduction

Global extremal functions were first introduced by J. Siciak [Sic1], in the spirit of the classical work of F. Leja [Lej], in order to extend classical results of approximation and interpolation to holomorphic functions of several complex variables. Later V. P. Zahariuta [Za2] gave another definition of the global extremal function based on the following class of plurisubharmonic functions on $\mathbf{C}^N$:

$$\mathcal{L}(\mathbf{C}^N) := \{v \in \mathrm{PSH}(\mathbf{C}^N) : \exists c_v \in \mathbf{R},\, v(z) \leqslant c_v + \log^+|z|,\, \forall z \in \mathbf{C}^N\}. \tag{1.1}$$

This class is called the class of plurisubharmonic functions of logarithmic growth (or minimal growth) on $\mathbf{C}^N$.

Then given a compact set $K \subset \mathbf{C}^N$, we define its global extremal function on $\mathbf{C}^N$ by the formula

$$L_K(z) = L_K(z; \mathbf{C}^N) := \sup\{v(z) : v \in \mathcal{L}(\mathbf{C}^N),\, v|K \leqslant 0\}, \quad z \in \mathbf{C}^N. \tag{1.2}$$

It has been proved by Siciak that the function $L_K$ is locally bounded on $\mathbf{C}^N$ if and only if $K$ is nonpluripolar in $\mathbf{C}^N$. In this case, the upper semi-continuous regularization $L_K^*$ of the function $L_K$ in $\mathbf{C}^N$ belongs to the class $\mathcal{L}(\mathbf{C}^N)$ (see [Sic2], [Kl]). Moreover, if $U \Subset \mathbf{C}^N$ is a domain and $K \subset U$ is a nonpluripolar compact subset, then the following fundamental inequality, known as the *Bernstein–Walsh inequality*, holds: there exists a constant $R = R(K; U) > 1$ such that

$$\|f\|_U \leqslant \|f\|_K R^d, \quad \forall f \in \mathcal{P}_d(\mathbf{C}^N),\, \forall d \geqslant 1, \tag{1.3}$$

where $\mathcal{P}_d(\mathbf{C}^N)$ is the space of holomorphic polynomials on $\mathbf{C}^N$ of degree at most $d$. It is known ([Sic2]) that the best constant $R := R(K; U)$ in the inequality (1.3) is related to