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Continuous analogues of Fock space IV: essential states

by

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1. Introduction

In this paper, the last of our series [1], [4], [5], we present a new procedure for constructing examples of E_0-semigroups, and we show how these methods can be applied to settle a number of problems left open in [1], [4] and [5]. The central objects of study are semigroups $\alpha=\{\alpha_t: t\geqslant 0\}$ of normal *-endomorphisms of the algebra $\mathcal{B}(H)$ of all operators on a (separable) Hilbert space H, which are continuous in the sense that $\langle \alpha_t(A)\xi, \eta\rangle$ should be a continuous function of t for fixed A in $\mathcal{B}(H)$ and fixed ξ, η in H.

α is called an *E_0-semigroup* [11] if it is unital in the sense that $\alpha_t(1)=1$, for every $t\geqslant 0$. At the opposite extreme, α is called *singular* if the projections $P_t=\alpha_t(1)$ decrease to zero as $t\to\infty$. While it is E_0-semigroups that are of primary interest, much of our analysis will concern singular semigroups. In particular, we will show that the generator of a singular semigroup is injective, and that its inverse is an unbounded completely positive linear map which can be represented in very explicit terms. Perhaps surprisingly, the results of this analysis of singular semigroups can be applied directly to E_0-semigroups. This is accomplished by making appropriate use of the *spectral C^*-algebra* $C^*(E)$ associated with a product system E ([4], [6]).

Recall that a *product system* is a measurable family of Hilbert spaces $E=\{E_t: t>0\}$ over the open interval $(0,+\infty)$, on which there is defined a (measurable) associative

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