

Derivative estimates for the Navier–Stokes equations in a three dimensional region

by

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Chapter I. The Navier–Stokes equations and the scalar viscosity potential

1. Introduction

The mathematical theory of the Navier–Stokes equations has centered upon basic questions of the existence, uniqueness, and regularity of solutions of the initial value