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Relative K-theory and topological cyclic homology

by

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0. Introduction

Let $f: A \to B$ be a map of rings up to homotopy (or "FSP's", see e.g. [Bö1], [Bö3] or Definition 3.1 below). When is it possible to give a good description of the relative algebraic K-theory? Generally, K-theory is hard to calculate, so it is of special importance to be able to measure the effect of a change of input.

Special instances of the case where f induces an epimorphism $\pi_0(A) \to \pi_0(B)$ with nilpotent kernel have been studied by several authors. The first general result in this direction was Goodwillie's theorem [G1], that in the case of simplicial rings, relative K-theory is rationally given by the corresponding relative negative cyclic homology. Recently, McCarthy has complemented this by giving a short and beautiful proof [Mc] showing that at a given prime p, the relative K-theory is given by the corresponding relative topological cyclic homology.

Although of great interest, simplicial rings do not cover all the important cases. In algebraic K-theory of spaces (Waldhausen's A-theory) the same question has been given considerable attention. If $X \to *$ is a 2-connected map (corresponding to a 1-connected map of FSP's), it is shown in [Bö2] that at a given prime p the relative K-theory is given by the topological cyclic homology. In [G5, p. 621] Goodwillie announced the statement for general 2-connected maps $X \to Y$ (this will also follow from the main theorem below).

This paper stemmed from a desire to understand the linearization $A(BG) \to K(\mathbf{Z}[G])$ (which corresponds to a 1-connected map of FSP's); that is, the connection between the algebraic K-theory of spaces and the algebraic K-theory of rings, each of which has theorems of the desired sort. Waldhausen has shown that this map is a rational equivalence, but torsion information has so far been out of reach.

In this paper we prove the conjecture of Goodwillie, posed at the International Congress of Mathematicians in Kyoto, 1990 [G5, p. 628].

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