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Relative algebraic K-theory and topological cyclic homology

by

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Introduction

In recent years, the study of the algebraic K-theory space $K(R)$ of a ring R has been approached by the introduction of spaces with a more homological flavor. One collection of such spaces is connected to K-theory by various trace maps and is particularly effective in measuring the relative K-theory $K(f)$ associated to a surjective ring homomorphism $f: R \to S$ whose kernel is nilpotent. Goodwillie's main theorem in [10] shows that the rational homotopy type of $K(f)$ can be recovered from cyclic homology. The main theorem of this paper shows that, for any prime p, the p-adic homotopy type of $K(f)$ can be recovered from topological cyclic homology, $TC(f)$.

To define the topological cyclic homology $TC(R)$ for a ring R one must first consider ordinary rings as special types of more general rings up to homotopy. Functors with smash product, or FSP's, were introduced by M. Bökstedt in [2] as useful models for such topological rings and it is for these objects that topological cyclic homology is defined by Bökstedt–Hsiang–Madsen in [4]. Every ring naturally gives rise to an FSP which models the associated Eilenberg–MacLane ring spectrum of the ring, and in this way rings and simplicial rings are naturally embedded into the category of FSP's. One can extend the definition of algebraic K-theory from simplicial rings to FSP's so that the algebraic K-theory of a simplicial ring agrees with the algebraic K-theory of its associated FSP. There is a natural transformation $\mathrm{trc}: K \to TC$, called the cyclotomic trace, which was used by Bökstedt–Hsiang–Madsen in [4] to solve the algebraic K-theory analogue of Novikov's conjecture for a large class of discrete groups. In [12], Goodwillie conjectured that for maps f of FSP's such that $\pi_0(f)$ (a ring map) is surjective with nilpotent kernel then the relative cyclotomic trace from $K(f)$ to $TC(f)$ would be an equivalence after pro-finite

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