

# The second main theorem for small functions and related problems

by

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## 1. Introduction

### 1.1. Results

One of the most interesting results in value distribution theory is the defect relation obtained by R. Nevanlinna: If  $f$  is a non-constant meromorphic function on the complex plane  $\mathbf{C}$ , then for an arbitrary collection of distinct  $a_1, \dots, a_q \in \mathbf{P}^1$ , the following defect relation holds:

$$\sum_{i=1}^q (\delta(a_i, f) + \theta(a_i, f)) \leq 2. \quad (1.1.1)$$

Here, as usual in Nevanlinna theory, the terms  $\delta(a_i, f)$  and  $\theta(a_i, f)$  are defined by

$$\delta(a_i, f) = \liminf_{r \rightarrow \infty} \left( 1 - \frac{N(r, a_i, f)}{T(r, f)} \right),$$
$$\theta(a_i, f) = \liminf_{r \rightarrow \infty} \frac{N(r, a_i, f) - \bar{N}(r, a_i, f)}{T(r, f)},$$