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# Complex geometry of convex domains that cover varieties

by

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## 1. Introduction

The main result of this paper is:

THEOREM 1. *Let $\Omega$ be a convex hyperbolic domain in $\mathbf{C}^n$ and suppose there is a subgroup $\Gamma\subset\mathrm{Aut}(\Omega)$ such that*

($\Gamma$1) $\Gamma$ *is discrete and acts freely* (*each $\gamma\in\Gamma$ is fixed-point free*),

($\Gamma$1) $\Gamma$ *is co-compact* (*in* $\Omega$).

*Then $\Omega$ is biholomorphic to a bounded symmetric domain.*

The hypothesis that $\Gamma$ acts freely can now be removed, see the comment on Lemma 11.8.

This confirms a conjecture cited by Yau in [36], p. 140. The hypotheses are equivalent to saying that there is a compact complex manifold $M$ whose universal cover is a convex hyperbolic domain $\Omega$ in $\mathbf{C}^n$. Thus, it is a type of uniformization theorem. Regarding the notion of hyperbolicity, see Proposition 2.8, and for a generalization weakening the convexity condition considerably see Theorem 2.6 which follows from Theorem 1 and the results in § 7.

The first part of this paper introduces a new method that given a non-compact automorphism group acting on a domain $\Omega$ produces continuous families of automorphisms. One needs some mild regularity hypothesis on the boundary, unless the automorphism group is co-compact, in this case convexity suffices, see Theorem 2.4. The general idea is to use boundary localization, involving rescaling, an idea used by Kuiper and Benzecri, in the context of affine and projective geometry, in the fifties,