

# A pointwise regularity theory for the two-obstacle problem

by

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## Introduction

A detailed study of the boundary regularity for solutions of the Dirichlet problem in an open region  $D$  of  $\mathbf{R}^N$ ,  $N \geq 3$ , was carried out by H. Lebesgue and others: this investigation culminated in the celebrated *Wiener criterion*. By relying on a fundamental notion of potential theory, namely that of *capacity* of an arbitrary subset of  $\mathbf{R}^N$ , N. Wiener was