Boundary behaviour of the complex Monge-Ampère equation

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1. Introduction

Following work by Yau [5] on the Calabi conjecture, Cheng and Yau [1] have shown that each smoothly bounded strictly pseudoconvex open set $\Omega\subset\mathbf{C}^n$, $n\geq 2$, admits a unique Kähler-Einstein metric equivalent to the Bergman metric. The condition that the metric be Einstein can be expressed as

$$R_{j\bar{k}} = -\partial_j\partial_{\bar{k}}(\log\det(G_{p\bar{q}})) = -(n+1)\,G_{j\bar{k}} \tag{1.1}$$

where $R_{j\bar{k}}$, $G_{j\bar{k}}$ are the components of the Ricci tensor and metric tensor respectively. The constant on the right-hand side could be any negative number; $-(n+1)$ is chosen for convenience.

One can search for such a metric by requiring that the potential $G\in C^\infty(\Omega)$ satisfy the following complex Monge-Ampère equation:

$$\det(\partial_j\partial_{\bar{k}}G) = e^{(n+1)G}. \tag{1.2}$$

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