A classification of Busemann G-surfaces which possess convex functions

by

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1. Introduction

A function φ defined on a complete Riemannian manifold M without boundary is said to be convex if φ is a one variable convex function on each arc-length parametrized geodesic. φ is locally Lipschitz continuous and hence continuous on M. It is a natural question to ask to what extent the existence of a convex function on M implies restrictions to the topology of M. In a recent work [4], the topology of M with locally nonconstant convex functions has been studied in detail. One of their results gives a classification theorem of 2-dimensional complete Riemannian manifolds which admit locally nonconstant convex functions: they are diffeomorphic to either a plane, a cylinder, or an open Möbius strip.

A classical result of Cohn-Vossen [3] states that a complete noncompact Riemannian 2-dimensional manifolds with nonnegative Gaussian curvature is homeomorphic to a plane, a cylinder, or an open Möbius strip. Moreover, Cheeger–Gromoll have proved in [2] that if a complete noncompact Riemannian manifold has nonnegative sectional curvature, then every Busemann function on it is convex (and locally nonconstant).

H. Busemann generalized Cohn-Vossen's result in [1] pp. 292–294, proving that a noncompact G-surface with finite connectivity and zero excess whose angular measure is uniform at π is topologically a plane, a cylinder, or a Möbius strip.

Now, the purpose of the present paper is to prove the following:

THEOREM 3.13. *Let R be a noncompact 2-dimensional G-space. If R admits a locally nonconstant convex function, then R is homeomorphic to either a plane, a cylinder $S^1 \times \mathbf{R}$, or an open Möbius strip.*

It should be noted that in the proof of the above result there is no analogy with the Riemannian case. This is because every point of a G-space R does not in general have