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The Newton polyhedron and oscillatory integral operators

by

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1. Introduction

The lack of suitable methods of stationary phase for both degenerate oscillatory integrals and degenerate oscillatory integral operators has been a major source of difficulties in many areas of analysis and geometry. Despite their name, degenerate phases are often generic, for example in presence of high codimension or of additional parameters, a phenomenon familiar in singularity theory. Strongest results to date include the now classic work of Varchenko [18] on decay rates for oscillatory integrals with generic analytic phases, and the relatively more recent progresses in the study of Lagrangians with Whitney folds [8], [4], [3], [9], generalized Radon transforms in the plane [10], [15], and sharp forms of the van der Corput Lemma in one dimension [11], [2].

The purpose of this paper is to establish sharp and completely general bounds for oscillatory integral operators on $L^2(\mathbf{R})$ of the form

$$(Tf)(x) = \int_{-\infty}^{\infty} e^{i\lambda S(x,y)} \chi(x,y) f(y)\, dy, \tag{1.1}$$

where $\chi \in C_0^\infty(\mathbf{R}^2)$ is a smooth cut-off function supported in a small neighborhood of the origin, and the phase $S(x,y)$ is real-analytic. (Besides its intrinsic interest, the decay rate of $\|T\|$ in $|\lambda|$ is closely related to the regularity of Radon transforms (see e.g. [3], [10], [15]), but we shall not elaborate on this point here.) Our main result is that the sharp bounds for $\|T\|$ as an operator on $L^2(\mathbf{R})$ are determined by the (reduced) Newton polyhedron of the phase $S(x,y)$. Remarkably, the Newton polyhedron is the notion which had been shown by Varchenko, confirming earlier hypotheses of Arnold, to control the

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