Acta Math., 179 (1997), 1–39

Hausdorff dimension and Kleinian groups

by

CHRISTOPHER J. BISHOP and PETER W. JONES

SUNY at Stony Brook
Stony Brook, NY, U.S.A.

Yale University
New Haven, CT, U.S.A.

1. Statement of results

Consider a group G of Möbius transformations acting on the 2-sphere S^2. Such a group G also acts as isometries on the hyperbolic 3-ball $\mathbf{B}$. The limit set, $\Lambda(G)$, is the accumulation set (on S^2) of the orbit of the origin in $\mathbf{B}$. We say the group is discrete if it is discrete as a subgroup of $\mathrm{PSL}(2,\mathbf{C})$ (i.e., if the identity element is isolated). The ordinary set of G, $\Omega(G)$, is the subset of S^2 where G acts discontinuously, i.e., $\Omega(G)$ is the set of points z such that there exists a disk around z which hits itself only finitely often under the action of G. If G is discrete, then $\Omega(G)=S^2\setminus\Lambda(G)$. G is called a Kleinian group if it is discrete and $\Omega(G)$ is non-empty (some sources permit $\Lambda=S^2$ in the definition of Kleinian group, but our results are easier to state by omitting it). The limit set $\Lambda(G)$ has either 0, 1, 2 or infinitely many points and G is called elementary if $\Lambda(G)$ is finite.

The *Poincaré exponent* (or *critical exponent*) of the group is

$$\delta(G)=\inf\left\{s:\sum_G \exp(-s\varrho(0,g(0)))<\infty\right\},$$

where ϱ is the hyperbolic metric in $\mathbf{B}^3$. A point $x\in\Lambda(G)$ is called a *conical limit* point if there is a sequence of orbit points which converges to x inside a (Euclidean) non-tangential cone with vertex at x (such points are sometimes called radial limit points or points of approximation). The set of such points is denoted $\Lambda_c(G)$. G is called *geometrically finite* if there is a finite-sided fundamental polyhedron for G's action on $\mathbf{B}$ and *geometrically infinite* otherwise. A result of Beardon and Maskit [6] says that G is geometrically finite if and only if $\Lambda(G)$ is the union of $\Lambda_c(G)$, the rank 2 parabolic fixed points and doubly cusped rank 1 parabolic fixed points of G. This makes it clear that $\dim(\Lambda_c)=\dim(\Lambda)$ and $\mathrm{area}(\Lambda)=0$ in the geometrically finite case.

The first author is partially supported by NSF Grant DMS-92-04092 and an Alfred P. Sloan research fellowship. The second author is partially supported by NSF Grant DMS-92-13595.