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Monstrous moonshine of higher weight

by

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1. Introduction

Suppose that V is a vertex operator algebra [B1], [FLM]. One of the basic problems is that of determining the so-called n-point correlation functions associated to V. There is a recursive procedure whereby n-point functions determine $(n+1)$-point functions [Z], so that understanding 1-point functions becomes important. In this paper we will study the 1-point functions on the torus associated with the moonshine module, which is of interest not only as an example of the general problem but because of connections with the monster simple group $\mathbf{M}$.

First we recall the definition of a 1-point function. Let the decomposition of V into homogeneous spaces be given by

$$V=\bigoplus_{n\geqslant n_0} V_n. \tag{1.1}$$

Each $v\in V$ is associated to a vertex operator

$$Y(v,z)=\sum_{n\in\mathbf{Z}} v(n)z^{-n-1} \tag{1.2}$$

with $v(n)\in\operatorname{End} V$. If v is homogeneous of weight k, that is, $v\in V_k$, we write $\operatorname{wt} v=k$. The *zero mode* of v is defined for homogeneous v to be the component operator

$$o(v)=v(\operatorname{wt} v-1), \tag{1.3}$$

and one knows that $o(v)$ induces an endomorphism of each homogeneous space, that is,

$$o(v)\colon V_n\to V_n. \tag{1.4}$$

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