Equivalent norms on Lipschitz-type spaces of holomorphic functions

by

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1. Introduction and results

A continuous function $\omega: [0, +\infty) \to \mathbb{R}$ with $\omega(0)=0$ will be called a majorant if $\omega(t)$ is increasing and $\omega(t)/t$ is nonincreasing for $t>0$. If, in addition, there is a constant $C(\omega)>0$ such that

$$
\int_0^\delta \frac{\omega(t)}{t} \, dt + \delta \int_\delta^\infty \frac{\omega(t)}{t^2} \, dt \leq C(\omega) \cdot \omega(\delta)
$$

whenever $0<\delta<1$, then we say that $\omega$ is a regular majorant. Given a majorant $\omega$ and a compact set $E \subseteq \mathbb{C}$, the (Lipschitz-type) space $\Lambda_\omega(E)$ consists, by definition, of the functions $f: E \to \mathbb{C}$ satisfying

$$
\|f\|_{\Lambda_\omega(E)} \triangleq \sup \left\{ \frac{|f(z_1) - f(z_2)|}{\omega(|z_1 - z_2|)} : z_1, z_2 \in E, z_1 \neq z_2 \right\} < \infty.
$$

Now let $D$ denote the unit disk $\{|z|<1\}$, $T$ its boundary and $\overline{D} \setminus T$. Further, let $A$ stand for the algebra of holomorphic functions on $D$ that are continuous up to $T$. We shall be concerned with the space

$$
A_\omega \triangleq A \cap \Lambda_\omega(\overline{D}),
$$

which in fact coincides with $A \cap \Lambda_\omega(T)$ (for regular majorants, this last statement follows from Lemma 4 below; for the general case, see [T]).

The purpose of this paper is to characterize the functions of class $A_\omega$ in terms of their moduli (the $\omega$'s involved are assumed to be regular majorants). To this end, we

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