THE MINIMUM OF A FACTORIZABLE BILINEAR FORM.

BY

E. S. BARNES
TRINITY COLLEGE, CAMBRIDGE.

1. Let

$$B(x, y, z, t) = (\alpha x + \beta y)(\gamma z + \delta t) \tag{1.1}$$

be a factorizable bilinear form, where $\alpha, \beta, \gamma, \delta$ are real, and x, y, z, t take all integral values subject to

$$xt - yz = \pm 1. \tag{1.2}$$

We suppose that $\Delta = \alpha\delta - \beta\gamma \neq 0$, and that B does not represent zero. Denoting the lower bound of $|B(x, y, z, t)|$ by $M(B)$, we have the following theorem, which is due to Davenport and Heilbronn[1]:

Theorem.

(i)
$$M(B) \leq \frac{3 - \sqrt{5}}{2\sqrt{5}} |\Delta|, \tag{1.3}$$

and equality occurs if and only if B is equivalent to a multiple of

$$B_1 = \left(x + \frac{1 + \sqrt{5}}{2} y\right)\left(z + \frac{1 - \sqrt{5}}{2} t\right), \tag{1.4}$$

in which case the lower bound is attained.

(ii) *For all forms not equivalent to a multiple of B,*

$$M(B) \leq \frac{2 - \sqrt{2}}{4} |\Delta|, \tag{1.5}$$

and equality occurs if and only if B is equivalent to a multiple of

---

[1] *Quarterly Journal* 18 (1947), 107—123.