THE CLOSEST PACKING OF CONVEX TWO-DIMENSIONAL DOMAINS.

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1. The following theorem is among the results proved by L. Fejes Tóth[1] in a recent paper.

Theorem. *Let $K_1, \ldots, K_n$ be n convex domains, which lie without mutual overlapping in a hexagon*[2] *H of area $a(H)$, and each of which arises from a given convex domain K by an area-preserving affine transformation. Then*

$$n h(K) \leq a(H), \tag{1}$$

where $h(K)$ denotes the area of the smallest hexagon circumscribed about K.

Some time ago I obtained a similar result on the restrictive hypothesis that the domains $K_1, \ldots, K_n$ are all congruent and similarly situated.[3] Although my results are largely superseded by those of Fejes Tóth, they are slightly stronger than his when the above condition is satisfied (especially when the domains do not have a centre of symmetry), and are obtained by a very different method. So I hope that the following statements of the results together with indications of the methods of proof may have some interest.

2. Let $\boldsymbol{a}, \boldsymbol{b}, \ldots, \boldsymbol{z}$ denote the points in two-dimensional space with coordinates $(a_1, a_2), (b_1, b_2), \ldots, (z_1, z_2)$; $\mathbf{0}$ being the origin with coordinates $(0, 0)$. Let

[1] Acta Sci. Math. (Szeged), 12 (1950), 62—67, see Theorem 1 and the remarks on page 66.

[2] A convex polygon having at most six sides will be called a hexagon.

[3] My first result, Theorem 2, was obtained in 1947, and was described in seminars in London, Cambridge, Bristol and Princeton in the years 1948—49; its most important consequence was announced in a paper by J. H. H. Chalk and myself (J. L. M. S., 23 (1948), 178—187 (179)). Detailed proofs of the results were given in the version of the present paper originally submitted to Acta mathematica.