NON-HOMOGENEOUS BINARY QUADRATIC FORMS.[1]

II. The second minimum of $(x + x_0)^2 - 7(y + y_0)^2$.

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Introduction.

1. In a previous paper[2] we showed that for all real x_0, y_0, there exist integers x, y for which

$$|f(x + x_0,\ y + y_0)| = |(x + x_0)^2 - 7(y + y_0)^2| \le \frac{9}{14}.$$

We further showed that for $(x_0, y_0) \not\equiv \left(\frac{1}{2}, \pm\frac{5}{14}\right)$, we can make $|f(x + x_0, y + y_0)|$ less than $\frac{1}{1.56}$. In this paper we modify our method with the help of a lemma, due to Dr. J. W. S. Cassels, to prove that the exact value of the second minimum is $\frac{1}{2}$. Our argument is purely geometrical and, as Dr. Cassels pointed out to the author, this seems to be the first time a purely geometrical argument has been applied to the study of the second minimum of a non-homogeneous form.

Our result can be stated as

Theorem: *Let $f(x, y) = x^2 - 7y^2$. Then, for any pair of real numbers x_0, y_0, there exist numbers x, y such that*

(1) $$x \equiv x_0 \ (\text{mod. } 1) \quad y \equiv y_0 \ (\text{mod. } 1)$$

and

(2) $$|f(x, y)| \le \frac{9}{14}.$$

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[1] This paper forms a part of author's thesis: Some Results in the Geometry of Numbers: approved for the degree of Ph.D. at the University of Cambridge.

[2] Non-homogeneous Binary Quadratic Forms (I): Acta mathematica, this vol. p. 1. We shall refer to this paper as NHF. For references also see NHF.