NON-HOMOGENEOUS BINARY QUADRATIC FORMS.[1]

I. Two Theorems of Varnavides.

By

R. P. BAMBAH

ST. JOHN'S COLLEGE, CAMBRIDGE.

Introduction.

1. Let $f(x, y)$ be an indefinite binary quadratic form $a x^2 + b x y + c y^2$, with positive discriminant $d = b^2 - 4ac$. A well-known theorem of Minkowski states that, for any real numbers x_0, y_0, there exist integers x, y such that

$$|f(x + x_0, y + y_0)| \leq \frac{1}{4}\sqrt{d},$$

the sign of equality being necessary if and only if $f(x, y)$ is equivalent to a multiple of xy.

Heinhold [1], Davenport [1], Varnavides [1] and Barnes [1] have found better estimates for the minimum for non-critical f.

Recently Davenport [2, 3, 4] studied the special forms $x^2 + xy - y^2$ and $5x^2 - 11xy - 5y^2$ and obtained interesting results about their minima. Varnavides [2, 3, 4] applied Davenport's method to the forms $x^2 - 2y^2$, $x^2 - 7y^2$, and $x^2 - 11y^2$. In this note we give straight-forward geometrical proofs of Varnavides' results about the forms $x^2 - 7y^2$ and $x^2 - 11y^2$.

The results we prove can be stated as

Theorem 1: *Let $f(x, y) = x^2 - 7y^2$. Then given any two real numbers x_0, y_0 we can find x, y such that*

$$x \equiv x_0 \pmod{1}, \; y \equiv y_0 \pmod{1} \tag{1.1}$$

and

$$|f(x, y)| \leq \frac{9}{14}. \tag{1.2}$$

[1] This note forms a part of author's thesis: Some Results in the Geometry of Numbers: approved for the degree of Ph.D. at the University of Cambridge.