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A compactification of Hénon mappings in $\mathbf{C}^2$ as dynamical systems

by

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1. Introduction

Let $H: \mathbf{C}^2 \to \mathbf{C}^2$ be a Hénon mapping

$$H\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} p(x)-ay \\ x \end{bmatrix}, \quad a \neq 0,$$

where p is a polynomial of degree $d \geqslant 2$, which without loss of generality we may take to be monic.

In [HO1], it was shown that there is a topology on $\mathbf{C}^2 \sqcup S^3$ homeomorphic to a 4-ball such that the Hénon mapping extends continuously. That paper used a delicate analysis of some asymptotic expansions, for instance, to understand the structure of forward images of lines near infinity. The computations were quite difficult, and it is not clear how to generalize them to other rational maps.

In this paper we present an alternative approach, involving blow-ups rather than asymptotics. We apply it here to Hénon mappings and their compositions, and in doing