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Quasi-isometric rigidity and Diophantine approximation

by

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1. Introduction

1.1. Background

A *quasi-isometry* between metric spaces is a map which only distorts distances by a bounded factor, above a certain fixed scale. (See §2.1 for a precise definition.) Each finitely generated group, equipped with a chosen finite generating set, has a natural *word metric*. In this metric, the group becomes a path metric space. Changing the generating set produces a new metric space, which is quasi-isometric to the original one. Thus, the quasi-isometric, or "large-scale geometric", structure of these metric spaces only depends on the group itself.

There has been considerable interest recently in understanding how the algebraic structure of a group influences its large-scale geometric structure, and vice-versa. (See [Gr] for a detailed survey.) Sometimes it happens that one can recover some, or all, of the algebraic structure of a group from its quasi-isometric properties. Broadly speaking, we call this phenomenon *quasi-isometric rigidity*.

Lattices in Lie groups provide a concrete and interesting family of finitely generated groups. A uniform—that is, co-compact—lattice in a Lie group is always quasi-isometric to the Lie group itself. In the co-compact case, then, the study of quasi-isometries of lattices reduces to the study of quasi-isometries of the ambient Lie group.

At least in the semisimple case, this theory is quite well developed. Quasi-isometries of the real hyperbolic space (plane) are just extensions of quasi-conformal (-symmetric) mappings. A similar, though perhaps less developed, theory holds in complex hyperbolic space. The result of [P] says that all quasi-isometries of quaternionic hyperbolic space (and the Cayley plane) are equivalent to isometries. (See §2.1 for the precise notion

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