Conformally natural extension of homeomorphisms of the circle

by

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1. Conformal naturality

Let G be the group of all conformal automorphisms of $D=\{z\in\mathbf{C};\ |z|<1\}$, and G_+ the subgroup, of index two in G, of orientation preserving maps. The group G_+ consists of the transformations

$$z\mapsto\lambda\frac{z-a}{1-\bar{a}z}$$

with $|\lambda|=1$ and $|a|<1$. For each such a, the map

$$g_a: z\mapsto\frac{z-a}{1-\bar{a}z} \tag{1.1}$$

in G_+ takes a into 0 and 0 into $-a$.

The group G operates on D, on $S^1=\partial D$, on the set $\mathcal{P}(S^1)$ of probability measures on S^1, on the vector space $\mathcal{T}(D)$ of continuous vector fields on D, etc. Explicitly

$$g\cdot z=g(z) \quad \text{if } z\in D\cup S^1,$$
$$(g\cdot\mu)(A)=g_*\mu(A)=\mu(g^{-1}(A)) \quad \text{if } \mu\in\mathcal{P}(S^1) \text{ and } A\subset S^1 \text{ is a Borel set},$$
$$(g\cdot v)(g(z))=g_*(v)(g(z))=v(z)g'(z) \quad \text{if } v\in\mathcal{T}(D),\ z\in D, \text{ and } g\in G_+,$$
$$(g\cdot v)(g(z))=g_*(v)(g(z))=\bar{v}(z)g'_{\bar{z}}(z) \quad \text{if } v\in\mathcal{T}(D),\ z\in D, \text{ and } g\in G\setminus G_+.$$

(We use the notations g'_z and $g'_{\bar{z}}$ for the complex derivatives of the function $g(z)$, and we

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