The surface C–C on Jacobi varieties and 2nd order theta functions

by

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Introduction

In their preprint [4], B. van Geemen and G. van der Geer stated four conjectures dealing with the modular significance of the surface C–C on a Jacobi variety. The first of these conjectures can be rephrased as follows:

(0.1) *Conjecture* ([4]). Let X be the jacobian of an irreducible non-singular algebraic curve C over $k=\mathbf{C}$, of genus $g\geq 1$. Let Γ_{00} be the vector space of sections of $\mathcal{O}_X(2\Theta)$ (Θ a symmetric theta divisor) having a zero of multiplicity at least 4 at $0\in X$, and write $F_X=\{x\in X|s(x)=0 \text{ for all } s\in\Gamma_{00}\}$. Then $F_X=\{x-y|x,y\in C\}$.

In loc. cit. the above authors give several partial results in this direction. Quite simultaneously, R. C. Gunning considered also this question in his paper [8], getting partial results, too (cf. also (2.1) below). Thirdly, in his bok [13], D. Mumford asked (we change some notations):

(0.2) *Question* ([13], p. 3.238). If D is a divisor class of degree 0 on C such that for all divisors E of degree $g-1$ for which $|E|$ is a pencil, then either $|D+E|\neq\emptyset$ or $|-D+E|\neq\emptyset$, then does it follow that $D\equiv a-b$ for some $a,b\in C$?

By standard reasons (cf. § 2), a positive answer to (0.2) would imply (0.1). (Actually, the answer to (0.2) is known to be negative if C is a trigonal curve.)

In this connection it is natural to ask also:

(0.3) *Question.* If D is a divisor class of degree 0 on C such that for all divisors E of degree $g-1$ for which $|E|$ is a pencil, then $|D+E|\neq\emptyset$, then does it follow that $D\equiv a-b$ for some $a,b\in C$?

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