On Dirichlet forms for plurisubharmonic functions

by

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§ 1. Introduction

We give a new proof of several basic properties of plurisubharmonic functions on $\mathbf{C}^n$ by making a systematic use of the notion of Dirichlet forms associated with closed positive currents of bidegree $(n-1, n-1)$. We further extend some of the properties stochastically and also exhibit some specific sample path behaviours of the related conformal diffusions. In the classical case that $n=1$, there are notions of the Laplace operator, the Green function, the Dirichlet integral and the Brownian motion, each of which is known to play an equivalent role to the subharmonic function in classical potential theory. In higher complex dimensions, we may think of the family of the above mentioned Dirichlet forms and the family of the conformal diffusions as the counterparts of the Dirichlet integral and Brownian motion respectively. Thus we may well expect that the Dirichlet space theory initiated by Beurling and Deny ([4], [8]) should work intrinsically in understanding and developing the theory related to the plurisubharmonic function.

First of all we describe the preliminary notions and notations. Let D be a bounded open set in the complex n-space $\mathbf{C}^n$. A function u on D taking values in $[-\infty, +\infty)$ is called *plurisubharmonic* (psh in abbreviation) if u is locally integrable on D with respect to the Lebesgue measure (denoted by V),

$$\sum_{\alpha,\beta=1}^{n} \frac{\partial^2 u}{\partial z_\alpha \partial \bar{z}_\beta} \xi_\alpha \bar{\xi}_\beta$$

is a positive distribution for any $\xi \in \mathbf{C}^n$ and

$$u(z) = \inf_{U(z)} V\text{-ess}\sup_{z' \in U(z)} u(z'), \quad z \in D,$$