The Laplacian for domains in hyperbolic space and limit sets of Kleinian groups

by

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1. Introduction and statement of results

Let X^{n+1} denote the real hyperbolic space of dimension $n+1$. We will make use of both the ball and upper half space models of X^{n+1}. The ball model is $B^{n+1}=\{x\in\mathbf{R}^{n+1};\ |x|<1\}$ with the line element $ds^2=4dx^2/(1-|x|^2)$. The upper half space model is $H^{n+1}=\{(x,y);\ x\in\mathbf{R}^n,\ y>0\}$ with the line element $ds^2=(dx^2+dy^2)/y^2$. When we write Δ, ∇ or dV, we are referring to the Laplacian, gradient and volume element, all with respect to the hyperbolic metric. For example in the H^{n+1} coordinates

$$dV=\frac{dx\,dy}{y^{n+1}}\quad\text{and}\quad -\Delta=y^2\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial x_1^2}+\ldots+\frac{\partial^2}{\partial x_n^2}\right)-(n-1)\,y\frac{\partial}{\partial y}.$$

Let Ω be an open connected subset of X^{n+1}; we denote by $W^1(\Omega)$ the space of functions

$$W^1(\Omega)=\{f\in L^2(\Omega);\ \nabla f\in L^2(\Omega)\}. \tag{1.1}$$

The quadratic forms H and D on $W^1(\Omega)$ are defined as

$$H(f,g)=\int_\Omega f\bar{g}\,dV,$$
$$D(f,g)=\int_\Omega \langle\nabla f,\overline{\nabla g}\rangle\,dV. \tag{1.2}$$

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