

Balls and metrics defined by vector fields I: Basic properties

by

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Introduction

In this paper we study the basic properties of certain balls and metrics that can be naturally defined in terms of a given family of vector fields. As an application, we then use these properties to obtain estimates for the kernels of approximate inverses of some non-elliptic partial differential operators, such as Hörmander's sum of squares. Some

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