Curvilinear enumerative geometry

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Contents

A good part of Enumerative Geometry, in its modern version, may be viewed as seeking to compute and "understand" fundamental classes of loci of configurations of figures, say points on a variety, satisfying natural geometric conditions. The difficulty of the problem often has much to do with the degenerate configurations, i.e. those whose points may coalesce in complicated ways. The *curvilinear* configurations are those which can degenerate at most like points on a smooth curve. The purpose of this paper is to develop a point of view, going back to Severi [23] and Le Barz [14], which leads to a solution of a good number of enumerative problems involving curvilinear configurations. This point of view consists in realizing natural loci of interest as *intersections*, in the following manner:

We are given an embedding $X\subset Z$; X_k or Z_k are suitable spaces parametrizing k-tuples on X or Z (which need not be precisely defined here), and $B^k\subset Z_k$ is a certain subspace, which should be thought of as well-understood and well-behaved. Then the locus of interest is the intersection.

$$X_k\cap B^k\subset Z_k.$$

Provided all these spaces can be reasonably defined, this viewpoint clearly shifts, in a