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A proof of Thurston's topological characterization of rational functions

by

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The criterion proved in this paper was stated by Thurston in November 1982. Thurston lectured on its proof on several occasions, notably at the NSF summer conference in Duluth, 1983, where one of the authors (JHH) was present. Using the notes of various attendants at these lectures, we have reconstructed a proof that we have made as precise as we could. Since this required a certain amount of work on our part, we thought it might be of some use to present this proof to the reader.

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After the first version was written, Clifford Earle pointed out that better estimates than what we had were to be found in [B].

Notations. $\#P$ = cardinality of P; $N=\{0,1,2,...\}$; $N^*=\{1,2,...\}$; P^1 = the Riemann sphere $\mathbf{C}\cup\{\infty\}$, i.e., the complex projective line.

1. Statement and definitions

Let $f: S^2 \to S^2$ be an orientation-preserving branched covering map. We denote by $\deg_x f$ the local degree of f at x. We will call

$$\Omega_f = \{x \mid \deg_x f > 1\}$$

the critical set of f, and

$$P_f = \bigcup_{n>0} f^n(\Omega_f)$$

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