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Physical measures for partially hyperbolic surface endomorphisms

by

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1. Introduction

In the study of smooth dynamical systems from the standpoint of ergodic theory, one of the most fundamental questions is whether the following preferable picture is true for almost all of them: The asymptotic distribution of the orbit for Lebesgue almost every initial point exists and coincides with one of the finitely many ergodic invariant measures that are given for the dynamical system. The answer is expected to be affirmative in general [14]. However, it seems far beyond the scope of present research to answer the question in the general setting. The purpose of this paper is to provide an affirmative answer to the question in the case of partially hyperbolic endomorphisms on surfaces with one-dimensional unstable subbundle.

Let M be the two-dimensional torus $\mathbf{T}=\mathbf{R}^2/\mathbf{Z}^2$ or, more generally, a region on the torus $\mathbf{T}$ whose boundary consists of finitely many simple closed C^2-curves: e.g. an annulus $(\mathbf{R}/\mathbf{Z})\times\left[-\frac{1}{3},\frac{1}{3}\right]$. We equip M with the Riemannian metric $\|\cdot\|$ and the Lebesgue measure $\mathbf{m}$ that are induced by the standard ones on the Euclidean space $\mathbf{R}^2$ in an obvious manner. We call a C^1-mapping $F\colon M\to M$ a *partially hyperbolic endomorphism* if there are positive constants λ and c and a continuous decomposition of the tangent bundle $TM=\mathbf{E}^c\oplus\mathbf{E}^u$ with $\dim\mathbf{E}^c=\dim\mathbf{E}^u=1$ such that

(i) $\|DF^n|_{\mathbf{E}^u(z)}\|>\exp(\lambda n-c)$;

(ii) $\|DF^n|_{\mathbf{E}^c(z)}\|<\exp(-\lambda n+c)\|DF^n|_{\mathbf{E}^u(z)}\|$

for all $z\in M$ and $n\geqslant 0$. The subbundles $\mathbf{E}^c$ and $\mathbf{E}^u$ are called the central and unstable subbundle, respectively. Notice that we do not require these subbundles to be invariant in the definition, though the central subbundle $\mathbf{E}^c$ turns out to be forward invariant from the condition (ii). The totality of partially hyperbolic C^r-endomorphisms on M is an open subset in the space $C^r(M,M)$, provided $r\geqslant 1$.