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Zeros of the i.i.d. Gaussian power series: a conformally invariant determinantal process

by

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1. Introduction

Consider the random power series

$$f_{\mathbf{U}}(z)=\sum_{n=0}^{\infty} a_n z^n, \tag{1}$$

where $\{a_n\}_{n=0}^{\infty}$ are independent standard complex Gaussian random variables (with density $e^{-z\bar{z}}/\pi$). The radius of convergence of the series is a.s. 1, and the set of zeros forms a point process $Z_{\mathbf{U}}$ in the unit disk $\mathbf{U}$. Zeros of Gaussian power series have been studied starting with Offord [20], since these series are limits of random Gaussian polynomials. In the last decade, physicists have introduced a new perspective, by interpreting the zeros of a Gaussian polynomial as a gas of interacting particles, see Hannay [12], Lebœuf [15] and the references therein. Much of the recent interest in Gaussian analytic functions was spurred by the papers Edelman–Kostlan [9] and Bleher–Shiffman–Zelditch [4]. A fundamental property of $Z_{\mathbf{U}}$ is the invariance of its distribution under Möbius transformations that preserve the unit disk; see §2 for an explanation, and Sodin–Tsirelson [27] for references.

Our main new discovery is that the zeros $Z_{\mathbf{U}}$ form a *determinantal process*, and this yields an explicit formula for the distribution of the number of zeros in a disk. Furthermore, we show that the process $Z_{\mathbf{U}}$ admits a conformally invariant evolution which elucidates the repulsion between zeros.

Given a random function f and points $z_1, ..., z_n$, let $p_\varepsilon(z_1, ..., z_n)$ denote the probability that for all $1\leqslant i\leqslant n$, there is a zero of f in the disk of radius ε centered at z_i. The

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