

The analogue of Picard's theorem for quasiregular mappings in dimension three

by

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1. Introduction

The theory of quasiregular mappings has turned out to be the right extension of the geometric parts of the theory of analytic functions in the plane to real n -dimensional space. The study of these mappings was initiated by Rešetnjak around 1966 and his main contributions to the theory is presented in the recent book [8]. For the basic theory of quasiregular mappings we refer to [2], [3], [13]. The definition is given in Section 2.1. In 1967 Zorič [14] raised the question of the validity of a Picard's theorem on omitted values for quasiregular mappings. Such a theorem appeared in 1980 in the following form.

THEOREM 1.1. [9]. *For each $K \geq 1$ and integer $n \geq 3$ there exists an integer $q = q(n, K)$ such that every K -quasiregular mappings $f: \mathbf{R}^n \rightarrow \mathbf{R}^n \setminus \{u_1, \dots, u_q\}$ is constant whenever u_1, \dots, u_q are distinct points in \mathbf{R}^n .*

Already from the early beginning of the theory it has been conjectured that the Picard's theorem is true in the same strong form for $n \geq 3$ as in the plane, namely that q can be taken to be 2 in Theorem 1.1. The purpose of this paper is to give a solution to this question in dimension three. The result is presented in Theorem 1.2. It shows that the conjecture is false and that Theorem 1.1 is indeed qualitatively best possible for $n=3$.

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