ELLIPTIC SYSTEMS IN $H_{s,\delta}$ SPACES ON MANIFOLDS WHICH ARE EUCLIDEAN AT INFINITY

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1. Introduction

In the subject of global analysis, there is a wealth of results in the case of a compact manifold which do not depend on the choice of a riemannian structure on the manifold, but in the non-compact case much less is known and moreover the results depend on the choice of a riemannian structure.

In this paper we study elliptic differential systems of order m on non-compact manifolds which are euclidean at infinity, in weighted Sobolev spaces $H_{s,\delta}$. Such a study has been done in weighted Hölder spaces $C_\beta^{1,\alpha}$, for equations of order 2 in [4]. On the other hand, M. Cantor has proved [2] closed range and isomorphism theorems for elliptic operators of order m in $\mathbf{R}^n$, in weighted Sobolev spaces $W_{s,\delta}^p$, where $p>n/(n-m)$. His paper is based on a work by L. Nirenberg and H. Walker [14] on the null spaces of such operators with continuous coefficients. In the present article we show that this restriction on p is unnecessary. Although we shall treat explicitly only the case $p=2$ which is of special interest since $W_{s,\delta}^2=H_{s,\delta}$ is a Hilbert space, the results extend trivially to any $p>1$. The hypotheses on the coefficients which we make, permit the study of nonlinear systems in the same framework.

Our exposition is self-contained, except in as far as it requires knowledge of local elliptic theory and results proved in [14] for operators with continuous coefficients on $\mathbf{R}^n$. The method relies on an improvement, given in § 2, of the imbedding theorem and multiplication lemma for the $W_{s,\delta}^p$ spaces. This improvement allows us to have $\delta>-n/p$ instead of $\delta\geqslant 0$ as in [2]. In § 3 some of the elliptic estimates on a compact manifold, with or without boundary, are recalled. In § 4 we extend the elliptic theory on $\mathbf{R}^n$ of [14] to operators with coefficients in the spaces H_{s_k,δ_k}. In § 5 we derive an isomorphism theorem for