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Representation theoretic rigidity in PSL(2, R)

by

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1. Introduction

Let G be a connected simple Lie group with trivial center, let Γ be an abstract group, and let ι_1 and ι_2 be inclusions of Γ in G. Assume throughout that each of the images $\iota_j(\Gamma)$ is a *lattice* subgroup, meaning that $\iota_j(\Gamma)$ is discrete and that the G-invariant measure on $G/\iota_j(\Gamma)$ has total finite mass. We say that ι_1 and ι_2 are *equivalent* if there is some automorphism ϱ of G so that $\iota_2=\varrho\circ\iota_1$. If G is not isomorphic to $\mathrm{PSL}(2,\mathbf{R})$ then the Mostow rigidity theorem (see [18], [19], [16] and [24]) says that ι_1 and ι_2 are necessarily equivalent. Alternatively, this says that any isomorphism between lattice subgroups of G extends to an automorphism of the whole group. This remarkable result fails for $\mathrm{PSL}(2,\mathbf{R})$ (see Section 2). Nonetheless, taking $G=\mathrm{PSL}(2,\mathbf{R})$, we have

THEOREM 1. *Suppose that π_1 and π_2 are irreducible unitary representations of* $\mathrm{PSL}(2,\mathbf{R})$, *not in the discrete series. Then $\pi_1\circ\iota_1$ and $\pi_2\circ\iota_2$ are equivalent representations of Γ if and only if ι_1 and ι_2 are equivalent inclusions and π_1 and π_2 are equivalent representations of* $\mathrm{PSL}(2,\mathbf{R})$.

As usual, two unitary representations of a group are called equivalent if there is a unitary equivalence of the two representation spaces which intertwines the two group actions. The situation is entirely different for discrete series representations, as explained in Section 8. Theorem 1 for $\iota_1\sim\iota_2$ was proven in [6].

The central step in the proof of Theorem 1 is a certain analytic criterion for the equivalence of ι_1 and ι_2. Let $\mathrm{PSL}(2,\mathbf{R})$ act on the upper half plane $\mathbf{H}=\{\mathrm{Im}(z)>0\}$ via

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}(z)=\frac{az+b}{cz+d},$$

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