

Calibrations and spinors

by

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Introduction

On a Riemannian manifold a calibration ϕ is simply a smooth p -form closed under exterior differentiation which is less than or equal to the volume form induced on each oriented p -dimensional submanifold. Each calibration determines a geometry of distinguished submanifolds, namely those submanifolds for which ϕ is exactly the induced volume form. The fundamental result of the theory of calibrations says that each closed submanifold, distinguished by a calibration, is automatically homologically volume minimizing. See the papers Harvey–Lawson [4] and Harvey [2], [3] for more details.

The purpose of this paper is first, to establish a general procedure for constructing (constant coefficient) calibrations by squaring spinors, and second, to use this general procedure to explicitly calculate new calibrations in sixteen variables—and hence new geometries of submanifolds.