ON CERTAIN EXTREMUM PROBLEMS FOR ANALYTIC FUNCTIONS.

By

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Introduction.

0.1. In order to state, in their simplest form, the type of problems to be discussed, we suppose, first, that

$$f(z)=\sum_{0}^{\infty} a_k z^k \tag{0.1.1}$$

is regular for $|z|\le 1$; and that $\mathfrak{k}(z)$ is regular for $|z|\le 1$, except for a finite number of poles β_i with $|\beta_i|<1$. Then

$$J(f)=\frac{1}{2\pi i}\int_{|\zeta|=1} f(\zeta)\,\mathfrak{k}(\zeta)\,d\zeta \tag{0.1.2}$$

is the sum of the residues of $f(z)\,\mathfrak{k}(z)$ at the points β_i. If, for instance, $\mathfrak{k}(z)=\sum_{0}^{n} c_k z^{-(k+1)}$ then $J(f)=\sum_{0}^{n} c_k a_k$; if $\mathfrak{k}(z)=n!\,(z-\beta)^{-(n+1)}$, $|\beta|<1$, then $J(f)=f^{(n)}(\beta)$.

In these and similar cases it is a natural and important problem to determine, for a given 'kernel' $\mathfrak{k}(z)$, the precise sup $|J(f)|$ when the functions $f(z)$ vary inside a suitably given class: for instance, the class of all f with $|f|\le 1$ in $|z|\le 1$.

0.2. In a previous paper [M-R][2] A. J. Macintyre and one of the present authors studied such extremum problems for the following classes H_p: Let $1\le p\le\infty$. If $p<\infty$ then H_p denotes the class of all functions $f(z)$ regular in $|z|<1$ for which the mean values

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[1] Dr. SHAPIRO's contribution represents work done on a doctorate thesis at M. I. T. under an A. E. C. predoctoral fellowship.

[2] MACINTYRE and ROGOSINSKI, quoted as [M-R] throughout. Compare the list of references at the end of this paper.