AN EXTENSION OF THE RIEMANN MAPPING THEOREM.

By

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Introduction.

There are no limits known for the boundary conditions under which the Laplace equation

$$\Delta u = 0$$

admits a solution for a given region. The very richness of the potential theory and the great variety of its applications seem to prohibit general results in this direction. There are, however, reasons indicating that the Laplace equation would permit solution under a boundary condition

$$\Phi\left(u, \frac{\partial u}{\partial x_1}, \ldots, \frac{\partial^2 u}{\partial x_1^2} \ldots\right) = 0$$

that expresses a relation between u and a given sequence of its partial derivatives of a much more general kind than those treated in the classical theory. The present paper will deal with the simplest 2-dimensional version of a problem of the indicated kind, that is directly related to the Riemann mapping theorem.

In this introduction Ω and Ω' will denote two entire complex planes and the euclidean 4-dimensional space E will be considered as the product $\Omega \times \Omega'$. A point in E will be denoted by (w, w') where

$$w = u + i v, \quad w' = u' + i v'.$$

A "surface" S in E will be defined as the boundary of some open pointset $K \subset E$. To each function $f(z)$ holomorphic in $|z| \leq 1$ we assign the curve L_f described in the space E by the point

$$(w, w') = (f(z), f'(z)) \qquad \left(f' = \frac{d f}{d z}\right),$$