UNITARY REPRESENTATIONS OF GROUP EXTENSIONS. I

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Introduction

Let $\mathfrak{G}$ be a separable locally compact group. Continuing the convention adopted in our papers [11] and [12] we shall abbreviate the term "continuous unitary representation of $\mathfrak{G}$" to "representation of $\mathfrak{G}$". If $\mathcal{K}$ is a proper closed subgroup of $\mathfrak{G}$ whose representations are in a suitable sense "all known" one may pose the following two questions. (a) Which representations of $\mathcal{K}$ are the restrictions to it of irreducible representations of $\mathfrak{G}$? (b) Given such a representation of $\mathcal{K}$ how can one construct all irreducible representations of $\mathfrak{G}$ of which it is the restriction? When $\mathcal{K}$ is the identity subgroup question (a) has a trivial answer (apart from questions of dimension) and question (b) is essentially the same as that of determining all irreducible representations of $\mathfrak{G}$? However, for other choices of $\mathcal{K}$, questions (a) and (b) can furnish a useful breakdown of the problem of determining all irreducible representations of $\mathfrak{G}$ into two more accessible components. It is the primary purpose of this paper to discuss questions (a) and (b) and their application to the determination of the representations of $\mathfrak{G}$ in the special case in which $\mathcal{K}$ is normal. Actually we shall find it more convenient to deal with the slight variation in which we identify representations of $\mathcal{K}$ which are quasi equivalent in the sense defined on page 195 of [12].

For the special case in which $\mathcal{K}$ is not only normal but commutative and in which $\mathfrak{G}$ is a semi direct product of $\mathcal{K}$ and $\mathfrak{G}/\mathcal{K}$ this program has been carried out in outline in our paper

(1) In large part the material in this paper has been described in outline in each of the following: (a) Two lectures given in Paris in October 1954 under the auspices of the "Colloque Henri Poincaré". (b) A series of ten lectures on group representations given under the auspices of the Princeton University physics department and supported by the Eugene Higgins fund. (c) A course in group representations given during the 1955 summer quarter at the University of Chicago. (d) A paper presented by title at the 1955 summer meeting of the American Mathematical Society (Abstract 61-6-726 t). Mimeographed lecture notes of the University of Chicago course have been issued by the University of Chicago mathematics department and it is possible that the Centre Nationale des Recherches Scientifiques will publish a volume containing the texts of the lectures presented at the "Colloque Henri Poincaré".

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