

ON THE REGULARITY OF THE SOLUTIONS OF BOUNDARY PROBLEMS

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0. Introduction

As is well known, the application of operator theory or calculus of variations to the study of differential equations leads to the question whether the solutions thus obtained are sufficiently smooth to be solutions in a classical sense. Rather complete results are known concerning the regularity of the solutions in the interior of their domain of existence (cf. Hörmander [6], Malgrange [9] and the references given there). The regularity at the boundary of solutions of boundary problems has been far less studied, although quite recently very important progress has been made (cf. Browder [1], Gusev [4], Lopatinski [8], Morrey-Nirenberg [10], Nirenberg [11]). The purpose of this paper is to give a complete description of the boundary conditions which give rise to regularity at the boundary, in the special case where the coefficients of the differential operators considered (in the interior and in the boundary conditions) are constant and the boundary is plane. This case can be studied essentially in the same way as the corresponding problem of interior regularity was studied by Hörmander [5], but considerable technical difficulties are added.