FINDING A BOUNDARY FOR A HILBERT CUBE MANIFOLD

BY

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1. Introduction

In [21] Siebenmann considers the problem of putting a boundary on an open smooth manifold. A necessary condition is that the manifold have a finite number of ends, that the system of fundamental groups of connected open neighborhoods of each end be "essentially constant" and that there exist arbitrarily small open neighborhoods of ∞ homotopically dominated by finite complexes. When the manifold has dimension greater than five and has a single such end, there is an obstruction $\sigma(\infty)$ to the manifold having a boundary; it lies in $\tilde{K}_0\pi_1(\infty)$, the projective class group of the fundamental group at ∞. When the manifold does admit a connected boundary, and is therefore the interior of a compact smooth manifold, such compactifications are conveniently classified relative to a fixed one by certain torsions τ in Wh $\pi_1(\infty)$, the Whitehead group of $\pi_1(\infty)$. In other words, σ is the obstruction to putting a boundary on the manifold and τ then classifies the different ways in which this can be done. One can deal with manifolds having a finite number of ends by treating each one in the above manner.

In this paper we carry out a similar program for the problem of putting boundaries on non-compact Q-manifolds, where a *Q-manifold M* is a separable metric manifold modeled on the Hilbert cube Q (the countable-infinite product of closed intervals).[2] The first problem is to decide upon a suitable definition of a boundary for a Q-manifold; for example $B^n \times Q$ is a perfectly good Q-manifold and $(\partial B^n)\times Q$ has every right to be called its boundary, but unfortunately there exist homeomorphisms of $B^n \times Q$ onto itself taking $(\partial B^n)\times Q$ into its complement. To see this just write Q as $[0, 1]\times Q$ and note that there exists a homeomorphism of $B^n\times[0, 1]$ onto itself taking $(\partial B^n)\times[0, 1]$ into its complement. In the

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(2) It is, for example, conjectured that Q-manifolds are precisely those ANR's that *locally* are compact ∞-dimensional and homogeneous.