

ON SECTIONS OF SOME HOLOMORPHIC FAMILIES OF CLOSED RIEMANN SURFACES

BY

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1. Introduction.

The study of holomorphic sections of the Teichmüller curves $\pi_n: V(p, n) \rightarrow T(p, n)$ was initiated by John Hubbard [8] for the case $n = 0$. The existence of such sections would be important because each such section would allow us to choose a point on every Riemann surface of genus p in a way that depends holomorphically on the moduli. Unfortunately, Hubbard showed in [8] that π_0 has no holomorphic sections if $p \geq 3$.

In our paper [5] we studied the holomorphic sections of π_n for $n \geq 1$, but we were unable to obtain complete results. Now we are able to describe all the holomorphic sections of π_n for every genus $p \geq 2$. We also study sections of $\pi_0: V(p, 0) \rightarrow T(p, 0)$ over subspaces of $T(p, 0)$ that correspond to Riemann surfaces with automorphisms. We state our theorems in § 2, and prove them in §§ 5, 7, and 8. Since our proofs require some unfamiliar facts from Teichmüller theory, we develop the facts we need in §§ 3 and 4. Much material in these sections, especially in § 3, is expository in nature. Both of our main theorems have generalizations, which we give in §§ 10 and 11 with indications of their proofs. We have chosen to focus our attention in the body of the paper on the most important cases.

The remaining two sections of the paper deal with projections of norm one in certain Banach spaces. In § 6 we prove two general propositions about the existence of such projections. In § 9 we establish the non-existence of such projections in certain spaces of quadratic differentials. Most cases of Theorem 9.1 were proved already in [5] and [8], and we prove the remaining cases by the methods indicated in [5].

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