NONTRANSITIVE QUASI-ORBITS IN MACKEY'S ANALYSIS OF GROUP EXTENSIONS

BY

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This paper is dedicated to George Mackey on his sixtieth birthday

0. Introduction

G. W. Mackey developed a general method for analyzing the dual of a locally compact group G (always second countable) in terms of the dual of a closed normal subgroup N and the cocycle duals of subgroups of G/N, provided that the action of G on the dual of N is sufficiently regular [9]. In this case regularity means that every ergodic quasi-invariant measure under the action of G is concentrated on an orbit, which means that the associated quasi-orbit is transitive. The theory of virtual groups was introduced by Mackey for the purpose of dealing with the less regular case [11, 12]. Section 9 of this paper gives proofs that the theorems of section 8 of [9] remain valid in the more general setting. It should be remarked that this leaves work yet to be done before a complete understanding of the general case is achieved. For instance, one of the theorems establishes a one–one correspondence between part of the dual of G and the ω-dual of a certain virtual group for a certain cocycle ω, but an example due to C. C. Moore shows that the latter can be empty [1]. This example is discussed in section 10 of this paper, and shows that representations of virtual groups need not decompose into primary representations.

The organization of the paper is as follows. The first six sections deal with the machinery of inducing representations from one group action to another. More particularly, sections 1 and 2 give preliminary material on Hilbert bundles and bundle representations of groupoids. In section 3 this is used to define induced representations, and it is proved that the definition given is an extension of the definition for subgroups. One novelty here is the proof of Proposition 3.4, which uses no special choice of Radon–Nikodym derivatives. In section 4 a lemma needed in later sections is proved, concerning intertwining operators.

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