THE BOUNDARY CORRESPONDENCE UNDER QUASICONFORMAL MAPPINGS

BY

A. BEURLING and L. AHLFORS

in Princeton, N. J. (A. B.) and Cambridge, Mass. (L. A.)

1. Introduction

1.1. The dilatation $D \geqq 1$ of a differentiable topological mapping f: $(x,y) \to (u,v)$ between plane regions is determined by

$$D + \frac{1}{D} = \frac{u_x^2 + u_y^2 + v_x^2 + v_y^2}{|u_x v_y - u_y v_x|}. \tag{1}$$

Geometrically, D represents the ratio between major and minor axis of the infinitesimal ellipse obtained by mapping an infinitesimal circle of center (x,y). From this interpretation it is evident that the dilatation is unaffected by conformal mappings of the planes. Furthermore, a mapping f and its inverse f^{-1} have the same dilatation at corresponding points.

A mapping is said to be *quasiconformal* if D is bounded. The least upper bound of D is referred to as the *maximal dilatation*.

1.2. It is known that a quasiconformal mapping of $x^2 + y^2 < 1$ onto $u^2 + v^2 < 1$ remains continuous on the boundary.[1] Hence it induces a topological correspondence between the two circumferences. We shall be concerned with the problem of characterizing this correspondence by simple necessary and sufficient conditions. More generally, we shall look for conditions which are compatible with a mapping whose maximal dilatation does not exceed a given number $K > 1$.

In view of the invariance with respect to conformal mappings we may replace the disk by the upper half-planes $y > 0$ and $v > 0$, and we may assume that the points at ∞

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[1] L. Ahlfors, On quasiconformal mappings. *Journal d'Analyse Mathematique*, vol. 7.1 (1953/54).