OSCILLATION AND DISCONJUGACY FOR LINEAR DIFFERENTIAL EQUATIONS WITH ALMOST PERIODIC COEFFICIENTS[1]

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1. Introduction and résumé of results

Equations of the form

$$(r(x)y')' + K(x)y = 0, \tag{1}$$

where $r(x) > 0$ and $K(x)$ are real continuous functions on $-\infty < x < \infty$, are classified, by the behavior of their real solutions, as $(+)$-oscillatory or non-oscillatory. In the first instance one non-trivial (not identically zero), and thereby every, solution vanishes at arbitrarily large abscissas; in the second instance every non-trivial solution is non-vanishing for sufficiently large abscissas. A special instance of non-oscillation is the disconjugate case in which every (non-trivial) solution has at most one zero on $-\infty < x < \infty$. It is known that an equation of the form (1) is disconjugate if and only if there is a solution which is everywhere positive.

Our principal interest concerns the situation where $r(x) \equiv 1$ and $K(x) = -a + bp(x)$. Here (a,b) are real parameters and $p(x)$ is a real almost periodic function. We shall note, in this case, that non-oscillation and disconjugacy are coincident. Also we shall find that the domain D in the (a,b)-parameter plane, for which the corresponding equations are disconjugate, is closed and convex.

We generalize the theory of Hill's equation (in which $p(x)$ is periodic) but, of course, without using the Floquet representation, which is not applicable here. For example, interior to the disconjugacy domain D there is a basis of solutions each of which has an almost periodic logarithmic derivative. For the boundary of D the analysis is more com-

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