TAUBERIAN THEOREMS FOR MULTIVALENT FUNCTIONS

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1. Introduction and background

Let
$$f(z)=\sum_0^\infty a_n z^n \tag{1.1}$$

be regular in $|z|<1$. If $f(z)$ has bounded characteristic in $|z|<1$ then it follows from classical theorems of Fatou that the Abel limit

$$f(e^{i\theta})=\lim_{r\to 1} f(re^{i\theta}) \tag{1.2}$$

exists p.p. in θ. In particular this condition is satisfied if $f(z)$ is mean p-valent in $|z|<1$ for some p.

In this paper we investigate under what conditions the power series (1.1) is summable by a Cesàro mean or is convergent at those points $e^{i\theta}$ where the Abel limit exists. It is classical that the existence of a Cesàro sum for $f(e^{i\theta})$ always implies the existence of the Abel limit (1.2) and in fact the existence of an angular limit.

The above problem was recently investigated by G. Halász [3] for univalent functions and certain subclasses of these functions. We define the αth Cesàro sums by

$$\sigma_N^{(\alpha)}(\theta)=\binom{\alpha+N}{N}^{-1}\sum_{n=0}^{N}\binom{\alpha+N-n}{N-n}a_n e^{in\theta}, \tag{1.3}$$

where $\alpha>-1$. Then Halász proved the following results.

THEOREM A. *If f is univalent in $|z|<1$ and* (1.2) *holds then, if $\alpha>2$*

$$\sigma_N^{(\alpha)}(\theta)\to f(e^{i\theta}); \tag{1.4}$$

also
$$\sigma_N^{(1)}(\theta)=O(\log N),\qquad \text{as } N\to\infty. \tag{1.5}$$

It is a classical result that (1.4) implies that

$$|a_n|=o(n^\alpha),\qquad \text{as } n\to\infty. \tag{1.6}$$