

THE $\lambda(\phi^4)_2$ QUANTUM FIELD THEORY WITHOUT CUTOFFS

III. *The physical vacuum*

BY

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1. Introduction

In this series of papers we construct a quantum field theory model. This model describes a spin-zero boson field ϕ with a nonlinear ϕ^4 selfinteraction in two dimensional space time. The corresponding classical field equation is

$$\frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2 \phi}{\partial x^2} + m_0^2 \phi + 4 \lambda \phi^3 = 0. \quad (1.1)$$

The classical field ϕ is by definition a real valued function of x and t which is a solution to (1.1). The quantum field ϕ is also a function of x and t , but its values $\phi(x, t)$ are densely defined bilinear forms on some Hilbert space. The quantum field ϕ is a solution to (1.1),

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