

# THE INHOMOGENEOUS MINIMA OF BINARY QUADRATIC FORMS (I).

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1. Let  $f(x, y) = ax^2 + bxy + cy^2$  be an indefinite binary quadratic form with real coefficients and discriminant  $D = b^2 - 4ac > 0$ . For any real numbers  $x_0, y_0$  we define  $M(f; x_0, y_0)$  to be the lower bound of  $|f(x + x_0, y + y_0)|$  taken over all integer sets  $x, y$ . It is clear that if

$$x'_0 \equiv x_0, y'_0 \equiv y_0 \pmod{1} \tag{1.1}$$

then

$$M(f; x'_0, y'_0) = M(f; x_0, y_0). \tag{1.2}$$