SOME PROPERTIES OF CONTINUED FRACTIONS.

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§ 1. Introduction.

Let

$$\{a_1, a_2, \ldots\} \tag{1}$$

be an infinite normal continued fraction, $a_1, a_2, \ldots$ being integers with $a_1 \geqq 0$, $a_k \geqq 1$ $(k = 2, 3, \ldots)$.

The consecutive convergents of (1) are denoted by $\frac{P_0}{Q_0}, \frac{P_1}{Q_1}, \frac{P_2}{Q_2}, \ldots$, where $\frac{P_0}{Q_0}$ has the usual symbolic sense $\frac{1}{0}$, and where the irreducible fraction $\frac{P_k}{Q_k}$ $(k \geqq 1)$ has the value of the continued fraction $\{a_1, a_2, \ldots, a_k\}$. We have:

$$\begin{cases} P_n = a_n P_{n-1} + P_{n-2}\,(n \geqq 2), \quad P_1 = a_1, \; P_0 = 1; \\ Q_n = a_n Q_{n-1} + Q_{n-2}\,(n \geqq 2), \quad Q_1 = 1, \; Q_0 = 0; \\ P_n Q_{n-1} - P_{n-1} Q_n = (-1)^{n+1}\,(n \geqq 1). \end{cases} \tag{2}$$

For $a_{n+1} \geqq 2$ $(n \geqq 1)$ the fractions

$$\frac{b P_n + P_{n-1}}{b Q_n + Q_{n-1}} \qquad (b = 1, 2, \ldots, a_{n+1} - 1) \tag{3}$$

are the interpolated fractions of (1). For $b = 1$ and $b = a_{n+1} - 1$ the fractions (3) are the extreme interpolated fractions between $\frac{P_{n-1}}{Q_{n-1}}$ and $\frac{P_{n+1}}{Q_{n+1}}$. The following theorems are well-known [1]:

1. *Is* α *a positive irrational number, then each convergent* $\frac{P}{Q}$ $(Q \geqq 1)$ *of* $\alpha = \{a_1, a_2, \ldots\}$ *satisfies the inequality:*

$$\left|\alpha - \frac{P}{Q}\right| < \frac{1}{Q^2}. \tag{4}$$