Cyclic cohomology for one-parameter smooth crossed products

by

G. A. ELLIOTT — *University of Copenhagen, Copenhagen, Denmark*

T. NATSUME — *University of Copenhagen, Copenhagen, Denmark* and *Saitama University, Shimo-okubo, Urawa, Japan*

and

R. NEST — *University of Copenhagen, Copenhagen, Denmark*

Introduction

In [1] A. Connes proved that, for an arbitrary C^*-dynamical system $(A, \alpha, \mathbf{R})$, there is a natural isomorphism

$$\phi_\alpha\colon K_i(A) \to K_{i+1}(A \times_\alpha \mathbf{R}), \quad i \in \mathbf{Z}/2\mathbf{Z}.$$

He showed also that, given an α-invariant trace τ on A, with dual trace $\hat{\tau}$ on $A \times_\alpha \mathbf{R}$, the equality

$$\hat{\tau}(\phi_\alpha([u])) = \frac{1}{2\pi i}\,\tau(u^*\delta(u)) \qquad (*)$$

holds for any unitary u in the domain of the infinitesimal generator δ of α.

In the terminology of [2], the right hand side of the above equality is just the pairing of a unitary and a cyclic one-cocycle. Of course, $\hat{\tau}$ is a zero-cocycle. Therefore the above equality reveals a certain relation between cyclic cocyles on an algebra and those on its crossed product.

The purpose of the present paper is to construct a machine which makes precise the relation between the cyclic theory of an algebra and that of a one-parameter crossed product.

Given a Fréchet algebra $\mathscr{A}$ and a one-parameter group α of automorphisms of $\mathscr{A}$ satisfying certain smoothness conditions (see Section 2.1), a Fréchet algebra that we shall call the smooth crossed product $\mathscr{A} \times_\alpha \mathbf{R}$ can be defined. Our main result is as follows.